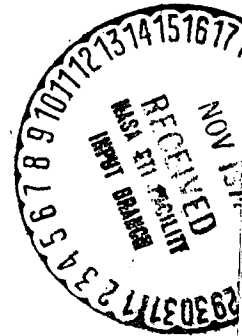


THE INFLUENCE OF ERRORS OF DISTANCE MEASUREMENT ON
THE PROBABILITIES OF LOCALIZATION

Centre National d'Etudes Spatiales

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THE INFLUENCE OF ERRORS OF DISTANCE MEASUREMENT ON
THE PROBABILITIES OF LOCALIZATION

Note:

It will be evident from this memorandum that the numerical simulation of the problem requires large-capacity computer facilities. The Automation Research Center has only a small computer available at present so that the simulation of all cases required can be made only with the assistance of the computer center of the National Space Research Center. For this reason only partial results have been presented here.

1. Principle of Measurements

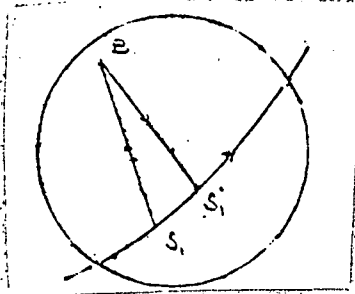
The investigation of the errors of localization in this memorandum is strictly limited to the case of localization-distance whose principal is resumed below.

The 400-Mc/s carrier is modulated by a subcarrier and a responder on the balloon retransmits at 400 Mc/s in phase-coherence over the subcarrier. The satellite measures the delay separating two given instants on the subcarrier, e.g.

$$\tau = \frac{n_1 d_1 + n_2 d'_1}{c}$$

where d_1 and d'_1 the roundtrip distance between satellite and balloon, and n_1 and n_2 are the indices of propagation. The balloon is consequently located on an ellipsoid of revolution with the foci S_1 and S'_1 corresponding to the instants of transmission

and reception from 2 to c tau



With satisfactory approximation, we can replace this ellipsoid by the sphere which has its center at the midpoint of S_1S_1' (subsequently designated by S_1) and a radius of $1/2 c \tau$ subsequently designated as d_1).

Under these conditions, the localization of the balloon requires two analogous measurements with the balloon situated at the intersection of three spheres (Note: Theoretically, these two measurements of distance will in general determine two points at the surface of the earth situated symmetrically in relation to the trace of the satellite on the earth. If the earth were not displaced in relation to the orbital plane, this ambiguity could not be solved in a single passage but terrestrial rotation makes it possible to overcome this difficulty) $S_1, d_1; S_2, d_2$: "sphere" of balloons. The error in the localization of the balloon may be due to:

- a) errors in measurement of distance ($d_1 d_2$);
- b) errors in the ephemerides (position S_1 and S_2 of the satellite);
- c) errors in the sphere of the balloons;
- d) errors in the altitude of the satellite.

We shall concentrate here specifically on errors of the first type.

2. Error in Measurement of Distance

The errors may be of several types:

a) Ionospheric Errors: The ionospheric error is relative and the electric trajectories S_1B and BS_1' can be assimilated to geometric distances. A study is actually under way at CNES for the purpose of evaluating this error, i.e. value, sign, variation.

b) Tropospheric and Atmospheric Errors: In order to minimize these, the possibility of measurement too close to the horizon is excluded. Consequently, this made it necessary to presuppose a certain elevation above the horizon on the order of $7^{\circ}5'$. As a first approximation, the change of the optical path to atmospheric refraction is, when expressed in meters, on the order of $4 \text{ cosec} \dots$.

Although the covering has been provided by taking this parameter into account, this does not exclude eventual interrogation at very low elevation and will raise the problem of the best manner of utilizing the interrogations during a single passage in order to reduce both ionospheric and atmospheric errors.

c) Electronic Errors: A study was undertaken at the request of CNES by the French Thomson-Houston Company ("result of measurements made on the model "Eole", October 1965). It appears from this study that the electronic errors in the measurement of distance can be considered as satisfying a standard law whose typical deviation obviously depends on the ratio between signal and noise. The most important findings are listed in table 1. (The typical deviation of distance is expressed in meters).

Table no. 1

Communication characteristics	in meters
No noise	150
Between satellite and balloon 12 dB Between balloon and satellite 11 dB	375
Between satellite and balloon 9 dB Between balloon and satellite 8 dB	500

Principle of the Study on Localization Errors for

3.1 - Earlier Findings

An initial study of localization errors was undertaken by CNES. The study was made on the hypothesis of symmetrical bases, i.e. the positions S_1 and S_2 of the satellite corresponding to the instant of interrogation t_1 and $t_1 + T$ are assumed as symmetrical in relation to the plane of the "inertial meridian" passing through the balloon where the "inertial" equator is the plane of the orbit (cf. fig. 1). In its large lines, the principle of the method can be resumed as follows:

If d is the distance between satellite and balloon, R is the radius of the earth, and R_s is the radius of the orbit assumed locally circular, we have

$$d^2 = R^2 + R_s^2 - 2RR_s \cos \alpha$$

$$\text{with } \cos \alpha = \cos \gamma \cos \Lambda_i$$

$$\text{and consequently } \cos \gamma = \cos \Lambda_i = \frac{R^2 + R_s^2 - d^2}{2RR_s}$$

Through differentiation we obtain:

$$\sin \varphi \cos \Lambda_i d\varphi - \sin \Lambda_i \cos \varphi d\Lambda_i = \delta R \left(\frac{b}{R} - \frac{1}{R_s} \right)$$

$$\delta R_s \left(\frac{b}{R} - \frac{1}{R_s} \right) = \frac{d\delta d}{RR_s} \quad (1)$$

For the two measurements made at S_1 and S_2 , we have two relations of the type of relation /illegible/ 1 which make it possible to determine the errors of longitude $d\Lambda_i$ and of

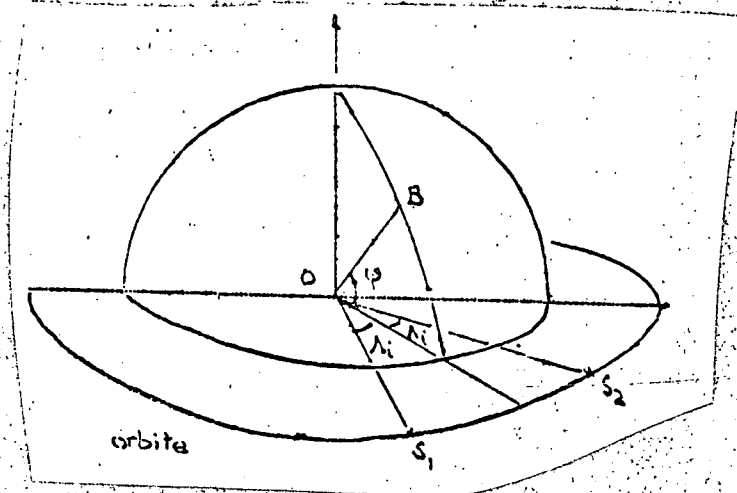


Fig.1

By designating as C_1 and C_2 the values of the second members of equ. (1) at S_1 and S_2 and on the hypothesis of symmetrical bases, we have

$$d\varphi = \frac{C_1 + C_2}{2 \cos \Lambda \sin \varphi}$$

$$d\Lambda = \frac{C_1 + C_2}{2 \sin \Lambda \cos \varphi}$$

In the specific case where we may assume that C_1 and C_2 have the same absolute value, we find that: if C_1 and C_2 have the sign, the error of longitude is 0 and the error of latitude is maximum and equal to

$$C / \cos \Delta \sin \varphi.$$

If C_1 and C_2 have opposite signs, the error of latitude is 0 and the error of longitude is maximum and equal to

$$C / \sin \Delta \cos \varphi.$$

In the specific case where the errors are only in the measurement of distance

$$C = \frac{d \delta d}{RR_s}$$

the maximum errors in both cases are

$$\varphi_1 = R \cos \varphi \, d\lambda = \frac{d \delta d}{R_s \sin}$$

$$\varphi_2 = R \, d = \frac{d \delta d}{R_s \cos \Delta \sin \Delta}$$

counted respectively on the "parallel" and on the "meridian" of B. Made by CNES for delta d = 1 km, the calculation of the functions P_1 and P_2 specifically shows that:

in regard to errors of longitude, the latter decrease very rapidly when we increase the distance of the bases where a distance of 20° leads to errors between 1.2 and 2.2 km as a function of longitude. In the case of closely adjacent bases, the error becomes very large when we move away from the trace.

In regard to the errors of latitude, the latter are little subject to the distance of the bases at high latitudes.

By contrast, in the neighborhood of the trace, the error of localization is large, regardless of spacing, and increases very rapidly with the latter.

On the whole, if the balloon is distant from the trace, a mean spacing of the bases makes possible an accurate localization and we shall have a certain latitude in selecting the rate of interrogation.

If the balloon is close to the trace, the localization will be in any event rather poor and it will be advantageous to adopt bases relatively close together.

3.2 - Generalization

3.2.1 - Description of Problem

The preceding results were obtained on the simplified basis of "symmetrical bases". No evaluation could be made of the consequences of this hypothesis on the validity of the results but it is clear in any event that, since the interrogation of the balloons is made in sequence, we will not have any practical possibility of adopting these conditions, however favorable. In addition, calculations were performed on the assumption that the errors made during the successive interrogations for localization were equal in absolute value which is a hypothesis far from true in a large number of cases.

The solution would therefore seem to reference ourselves more closely to the physical reality of the phenomenon and to define no longer the maximum errors but the probabilities of localization within a given accuracy. The actual problem may be conceived as follows:

A given balloon B on the surface of the earth will be at a certain instant within the field of visibility of the

satellite, i.e. it will be seen along a certain orbit, during the interval of time t_0 to t_n , S_0 and S_n corresponding to the start and end of visibility. If T is the time of interrogation, the first interrogation of the balloon B will take place at some instant T_1

$$t_0 < t_1 < t_0 + T$$

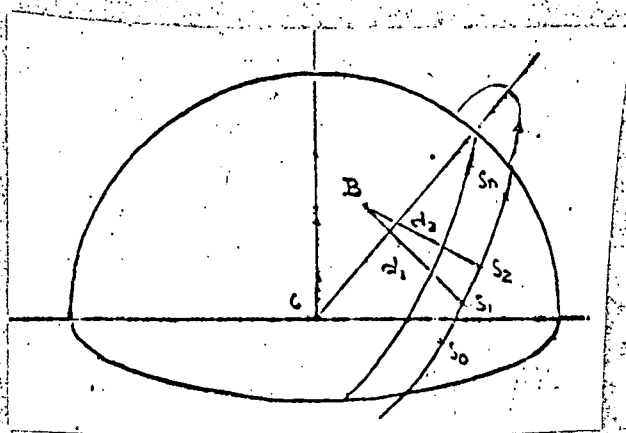
and the following interrogations at

$$t_1 + T \dots t_1 + nT$$

as long as

$$t_1 + nT < t_n$$

For each of the positions S_1 , S_2 and S_n , we know the "measured" distances $d_1^* d_2^* \dots$ which are the geometric distances $S_1 B_1$, $S_2 B_2 \dots S_n B_n$ (balloon-satellite) affected by a certain error.



Localization is derived from these measurements. It is evident that we must take into account at this stage: (1) the errors affecting the measurements, and (2) the terrestrial rotation during successive sequences of interrogation.

The second problem was treated in memorandum No. 5 and leads to a systematic error of localization which can be elim-

inated by a program of iterative calculation.

In regard to the first problem, we shall assume as hypothesis that the errors affecting the various measures satisfy a standard law of zero average from which we establish the typical deviation (Note: Properly speaking, deviation will be considered as a parameter of the system since it will depend on the technology of the available equipment. In the last analysis, the problem is precisely that of determining the accuracy of measurement required so that localization will be established at better than $\underline{x}$ km with a probability P in percent.

Strictly speaking, it appears difficult, even in a first phase, to assume that the wind will be zero which would mean that the succession of the balloons $B_1 B_2$ would be due exclusively to the rotation of the earth.

For example, in regard to distance, we must expect that the best localization will be established from rather far distant bases (orthonormal bases?) when there is no wind. The increase of T will lead in the case of wind, however, to an increase of the random distances $B_1 B_2$ which will make localization more difficult.

We must therefore expect that the probability of localizing a balloon at $\underline{x}$ km, as a function of the interval between interrogation T and in the event that any other parameter is the same, will be represented by the diagrams a) or b) of fig. 2 as a function of whether we assume wind as zero or not.

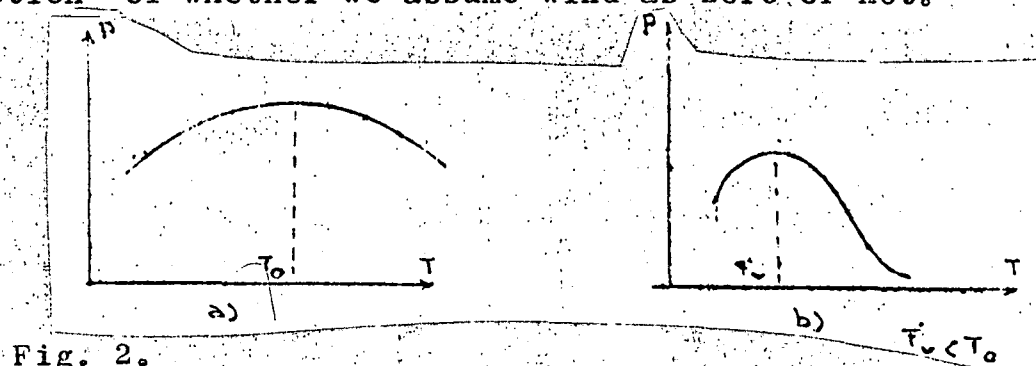


Fig. 2.

In spite of these restrictions and in view of the complexity of the programs, we made the assumption of zero wind as a first stage. The influence of the wind will be considered in Memorandum No. 6 /?/

3.2.2. - Formulation

As conceived on the basis of the physical description of the phenomenon, the computation program will be made in accordance with the following sequence in regard to the influence of errors of distance.

- 1 - We will establish the parameters of the orbit, i.e. altitude, inclination, interval between interrogation.
- 2 - We will establish a presumed position of the balloon characterized by the distance in relation to the trace of the satellite where the latter is a priori considered to be an important parameter.
- 3 - We determine the arc of visibility $S_0 S_n$ on the trajectory of the satellite corresponding to the instants $t_0 t_n$.
- 4 - Having established the successive positions of the satellite, we calculate the true distances satellite-balloon $B_1(S_1 B_1)$ $d_2(S_2 B_2)$ by taking into account, when applicable, the terrestrial rotation.
- 5 - In order to take into account errors of measurement, we generate the random numbers η_1 , η_2 corresponding to the estimated errors and also generate the "calculated" distances d_1^* , d_2^* . . . ($d_1^* = d_1 + \eta_1$).
- 6 - On the basis of d_1^* , we determine the "calculated" position of the balloon.
- 7 - We compare the true and calculated positions of the balloon and derive from them the deviation of distance.
- 8 - This deviation is quantified and filed./stored?/

9 - We repeat the operation a certain number of times (300) in order to obtain a histogram /?/ which will give the probability of localization at $\underline{x}$ km if the number of samplings is sufficient.

10 - We modify the parameters, either the sigma of the measures or the orbital parameters.

Execution of this program therefore supposes that we are able to solve the following problems: P (a) from the known position of the balloon, estimate the arc of visibility t_{on} corresponding to the limit positions $S_0 S_n$ of the satellite.

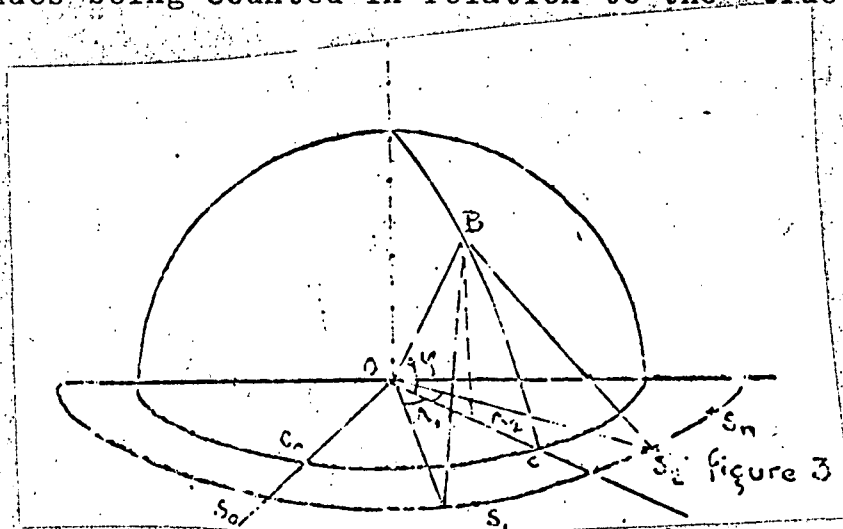
a) From the known positions of the balloon and the satellite at any given instant, calculate the true distances satellite-balloon.

c) From the known successive positions of the satellite $S_1 S_2$ and the measured distances " $d_1^* d_2^*$ ", localize the balloon.

d) Generate a sequence of random numbers of given distribution, typical deviation and given mean.

These problems will be considered in turn.

a) Visibility: We adopt the same representation as in para. 3.1 and take the orbital plane as "equatorial" plane with the longitudes being counted in relation to the "true balloon meridian."



The extreme positions S_0 S_n of the satellite visible from the balloon B correspond to the traces C_0 C_n such that the hemispheres centered on C_0 C_n (fo fig. 3) at an angle to the center O pass through B. We have:

$$\theta = \arccos (\beta \cos e) - e \quad \beta = R/R_s$$

$$\cos \gamma = \frac{\cos \theta}{\cos \beta}$$

$$\gamma = \widehat{C_0 C} = \frac{\Omega_s \gamma}{2}$$

which defines the time of visibility t_{ou} . The earth is here assumed as stationary because it was shown (of. memorandum no.5) that this effect could be taken accurately into account.

b) and c) Calculation of the true distances and localization:

We have

$$d_i^2 = R^2 + R_s^2 - 2RR_s \cos \gamma \cos \lambda_i$$

with

$$\lambda_{i+1} = \lambda_i + \Omega_s T$$

We stated

$$D_1 = \cos \gamma \cos \lambda_1$$

$$D_2 = \cos \gamma \cos (\lambda_1 + \Omega_s T)$$

and consequently

$$D_1^2 = R^2 + R_s^2 - 2RR_s D_1$$

$$d_2^2 = R^2 + R_s^2 - 2RR_s D_2$$

Knowing the true distances d_1 and d_2 of a base S_1 S_2 , we generate the "measured" distances

$$d^*_1 = d_1 + \gamma_1$$

$$d^*_2 = d_2 + \gamma_2$$

We then determine ϕ^* and λ^* "as calculated" by

$$D^*_1 = \cos \gamma^* \cos (\lambda_1 - \lambda^*) = (R^2 + R_s^2 - d^{*2}_1) / 2RR_s$$

$$D^*_2 = \cos \gamma^* \cos (\lambda_1 - \Omega_s T - \lambda^*) = (R^2 + R_s^2 - d^{*2}_2) / 2RR_s$$

or

$$\operatorname{tg} \lambda^* = \frac{D^*_2 \cos \lambda_1 - D^*_1 \cos (\lambda_1 + \Omega_s T)}{D^*_2 \sin - D^*_1 \sin (\lambda_1 + \Omega_s T)}$$

$$\cos \gamma^* = D^*_1 / \cos (\lambda^* - \lambda_1)$$

We then determine the error in $\underline{m}$ of longitude and latitude

$$R \cos \gamma \cdot \lambda^*$$

$$R (\gamma^* - \gamma) = R \Delta \gamma$$

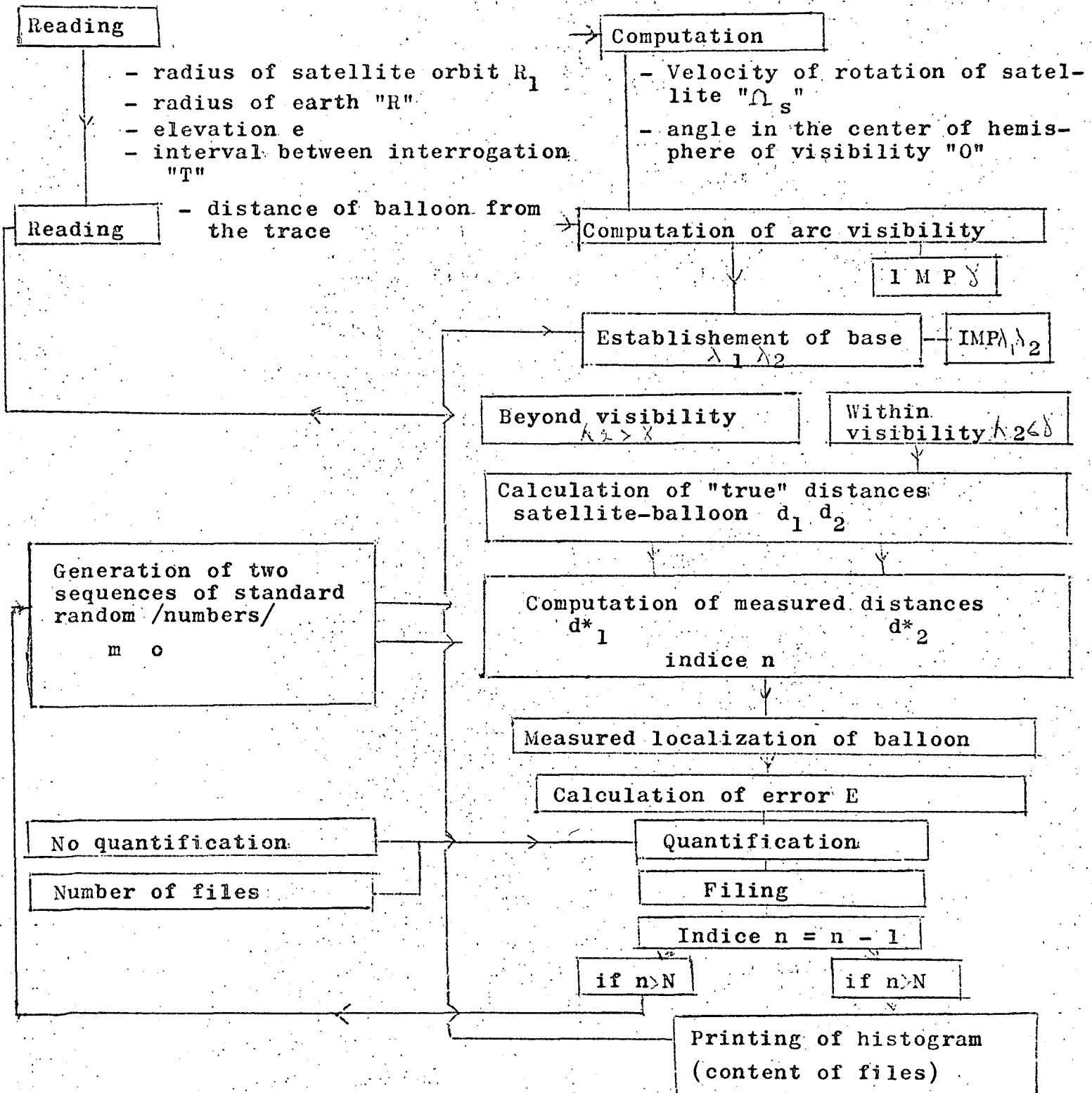
and consequently obtain the overall error of localization E

$$E^2 = R^2 \left[(\Delta \gamma)^2 + \cos^2 \gamma (\Delta \lambda)^2 \right]$$

The corresponding computation chart will be found on /p. 18 of source/.

d) Generation of standard random numbers of given typical deviation and mean value: (Cf. Annex) The computation programs and corresponding notations will be found in the Annex.

Computation diagram for distance localization.



$$\Omega_3 = \sqrt{398599 \frac{1}{R_s^3}}$$

$$\cos 0 = \cos^2 \varphi + \sin^2 \varphi \sqrt{1 - \cos^2 2}$$

$$\cos \gamma = \cos 0 / \cos \varphi$$

$$\lambda_2 = \lambda_1 + \Omega_s T$$

$$d_1^2 = R^2 + R_s^2 - 2RR_s \cos \varphi \cos \lambda_1$$

$$d_2^2 = R^2 + R_s^2 - 2RR_s \cos \varphi \cos \lambda_2$$

$$d^* = d + B$$

by stating

$$D^*_1 = (R^2 + R_s^2 - d^{*2}_1) / 2RR_s \quad \cos \varphi^* = D^*_1 / \cos (\theta^* - \lambda_1)$$

we get

$$D^*_2 = (R^2 + R_s^2 - d^2_2) / 2RR_s \quad \operatorname{tg} \theta = \frac{D^*_1 \cos (\lambda_2) - D^*_2}{D^*_2 \sin (\lambda_1) - D^*_1 \sin (\lambda_2)}$$

$$E = R \left[(\varphi^* - \varphi) - \theta^{*2} \cos^2 \varphi \right]^{1/2}$$

4 - Results

4.1 - Influence of Base Positions in Relation to the Balloon

4.1.1 - Optimum selection of bases: As in the preceding cases (rotating earth and wind), the calculations were performed by varying the base within the zone of visibility. Diagrams 1 and 2 furnish the probabilities of localization P_q at q km for an orbit of 900 km and an interval between interrogation of 195 secs for a balloon located at 5° or 20° from the trace.

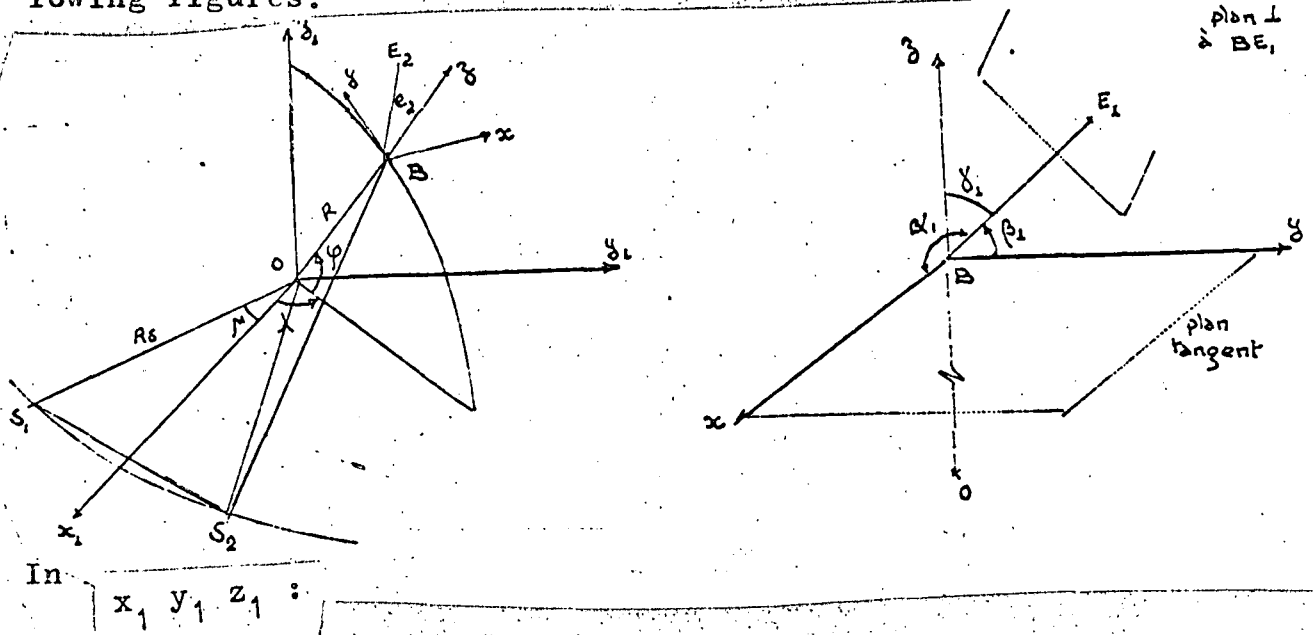
For $\sigma = 0.5$ km and in the case of a balloon close to the trace, the probability of localization at better than 2 km thus drops from 98 % in the case of symmetrical bases to less than 25 % at the limit of visibility. This phenomenon is general although it does not have the same importance at greater distance from the trace. For example, at a distance of 20°, the values found are respectively 85 and 60 %.

Let us consider the problem from the point of view of mean error. Let S_1, S_2 be the two positions of the satellite constituting the base and d_1, d_2 the true distances satellite-balloon possibly affected by the error e_1 and e_2 such that e_1, e_2 is part of a random Gauss sequence with

$$\overline{e_i} = 0$$

$$\overline{e_i^2} = \sigma^2$$

Since e_1 and e_2 are small in relation to R, d_1 , and d_2 , we subsequently merge the spheres $(S_1, d_1), (S_2, d_2)$ sphere of the balloons with their planes tangent to B. We then have the following figures:



$$S_1 \begin{vmatrix} R_s \cos \mu \\ -R_s \sin \mu \\ 0 \end{vmatrix} \quad S_2 \begin{vmatrix} R_s \cos \mu \\ R_s \sin \mu \\ 0 \end{vmatrix} \quad B \begin{vmatrix} R \cos \varphi \cos \lambda \\ R \cos \varphi \sin \lambda \\ R \sin \varphi \end{vmatrix}$$

On the other hand

$$B_x \begin{vmatrix} -\sin \lambda \\ \cos \lambda \\ 0 \end{vmatrix} \quad B_y \begin{vmatrix} -\sin \varphi \cos \lambda \\ \sin \varphi \sin \lambda \\ \cos \varphi \end{vmatrix} \quad B_z \begin{vmatrix} \cos \varphi \cos \lambda \\ \cos \varphi \sin \lambda \\ \sin \varphi \end{vmatrix}$$

and consequently

$$\begin{aligned} \overrightarrow{S_1} \cdot \overrightarrow{B} \cdot \overrightarrow{B_z} &= \cos \gamma_1 \cdot d_1 \cdot 1 = R - R_s \cos \varphi \cos (\lambda + \mu) \\ \cos \gamma_1 &= \frac{R - R_s \cos \varphi \cos (\lambda + \mu)}{d_1} \quad (=z) \end{aligned}$$

$$\begin{aligned} \text{and } \overrightarrow{S_1} \cdot \overrightarrow{B} \cdot \overrightarrow{B_x} &\rightarrow \cos \alpha_1 d_1 \cdot 1 = R_s \sin (\lambda + \mu) \\ \cos \alpha_1 &= \frac{R_s \sin (\lambda + \mu)}{d_1} \quad (=x) \end{aligned}$$

$$\begin{aligned} \overrightarrow{S_1} \cdot \overrightarrow{B} \cdot \overrightarrow{B_y} &\rightarrow \cos \beta_1 d_1 \cdot 1 = R_s \sin \varphi \cos (\lambda + \mu) \\ \cos \beta_1 &= \frac{R_s \sin \varphi \cos (\lambda + \mu)}{d_1} \quad (=y) \end{aligned}$$

We also have γ_2 , α_2 and β_2 by changing μ into $-\mu$, d_1 into d_2 : in xyz, we can replace the sphere (S_1, d_1) by the tangential plane:

The plane perpendicular to BE_S in $E_1(x_1 y_1 z_1)$ is written

$$(x-x_1) x_1 + (y-y_1) y_1 + (z-z_1) z_1 = 0$$

and consequently the intersection with $z = 0$ and also

$$\begin{array}{l|l} x_2; y_2 & x x_1 + y y_1 - e_1^2 = 0 \\ -x_1; -y_1 & x x_2 + y y_2 - e_2^2 = 0 \end{array}$$

and consequently the coordinate of E

$$x_E = \frac{y_2 e_1^2 - y_1 e_2^2}{(y_2 x_1 - x_2 y_1)} \quad \text{and} \quad y_E = \frac{x_2 e_1^2 - x_1 e_2^2}{(x_2 y_1 - x_1 y_2)}$$

with

$$x_1 = e_1 \cos \alpha_1$$

$$x_2 = e_2 \cos \alpha_2$$

$$y_1 = e_1 \cos \beta_1$$

$$y_2 = e_2 \cos \beta_2$$

$$z_1 = e_1 \cos \gamma_1$$

$$z_2 = e_2 \cos \gamma_2$$

We thus obtain

$$(y_2 x_1 - x_2 y_1) = \frac{R_s^2 e_1 e_2}{\sin \varphi \sin 2\mu}$$

$$x_E = \frac{e_1 d_1 \cos(\lambda - \mu) - e_2 d_2 \cos(\lambda + \mu)}{R_s \sin 2\mu}$$

$$y_E = - \frac{e_1 d_1 \sin(\lambda - \mu) - e_2 d_2 \sin(\lambda + \mu)}{R_s \sin \varphi \sin 2\mu}$$

Let us find the mean of the module of BE by taking into account

$$\bar{e}_1^2 = 0^2 = \bar{e}_2^2 \quad \text{and} \quad (\bar{e}_1 \bar{e}_2) = \bar{e}_1 \times \bar{e}_2 = 0$$

$$\bar{e}_1 = 0 \quad \bar{e}_2 = 0$$

(independent errors)

$$\bar{x}_E^2 = \frac{\sigma^2}{R_s^2 \sin^2 2\mu} \left[d_1^2 \cos^2(\lambda - \mu) + d_2^2 \cos^2(\lambda - \mu) \right]$$

$$\bar{y}_E^2 = \frac{\sigma^2}{R_s^2 \sin^2 \gamma \sin^2 \mu} \left[d_1^2 \sin^2(\lambda - \mu) + d_2^2 \sin^2(\lambda - \mu) \right]$$

from which

$$\bar{E}^2 = \bar{x}_E^2 + \bar{y}_E^2$$

with

$$d_1^2 = R^2 + R_s^2 - 2RR_s \cos \gamma \cos(\lambda - \mu)$$

$$d_2^2 = R^2 + R_s^2 - 2RR_s \cos \gamma \cos(\lambda - \mu)$$

We thus have the calculation program given in the Annex which is obtained by stating $u = R_s/R$.

a) In particular, we find the same results given by CNES for symmetrical bases. In that case we have

$$\lambda = 0 \quad d_1 = d_2 = d$$

$$\text{or} \quad y_E = \frac{d(e_1 - e_2)}{2R_s \sin \gamma \cos \mu} \quad x_E = \frac{d(e_1 - e_2)}{2R_s \sin \mu}$$

If we then assume $e_1 = e_2$, we obtain $x_E = 0$

$$y_E = \frac{d e}{R_s \sin \gamma \cos \mu} \quad \text{and the error is in the latitude.}$$

If $e_1 = -e_2$

$$x_E = \frac{d e}{R_s \sin \mu}, \quad y_E = 0$$

the error is in the longitude. If we attempt to find the equality condition of the above errors: we obtain $\sin \phi = \tan \mu$ (orthogonal bases).

The hypotheses of CNES consequently amounted to admitting for e_1 and e_2 the values $+1$ and -1 with equal probability.

b) The results are plotted in diagrams 4 and 5. It will be noted that the position of the base is critical, particularly in regard to short intervals between interrogation. In addition, the mean value of the error for a base half way between center and limit of visibility decreases if T increase does not bring about any large gain of accuracy.

If we consider the influence of the distance from the trace, we find that distance from the trace produces a flattening of the curves with increase of the error at the level of the centered bases.

c) The preceding results were obtained in the case of absolute errors with a Gauss distribution (white noise). This hypothesis has been justified earlier but it should be stressed that the above conclusions depend essentially on the law of probability selected.

d) In order to justify a better localization from orthonormal bases, diagram 6 gives the development of the mean error for different base centers at a larger scale.

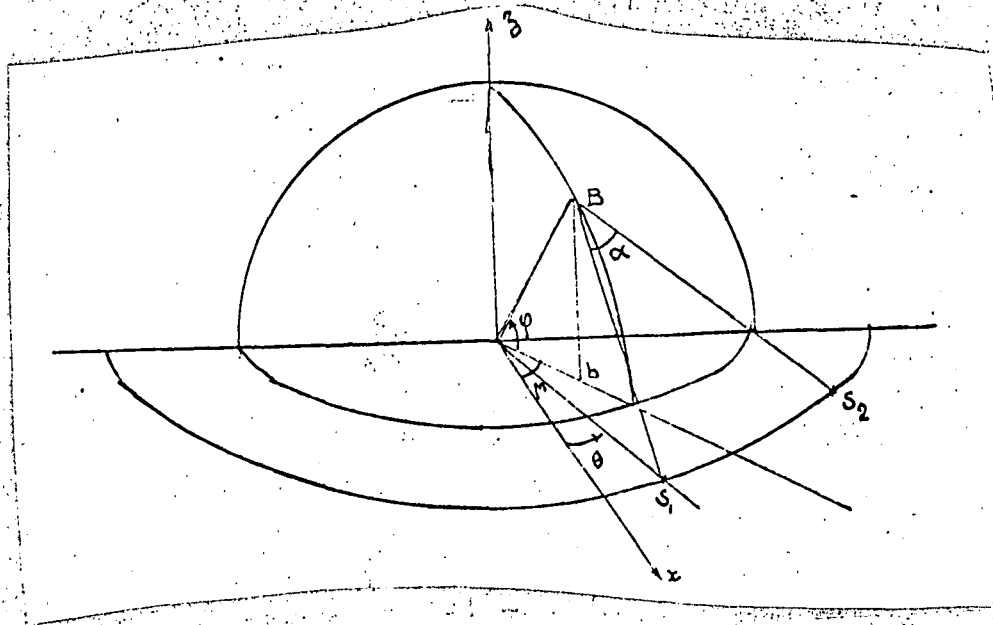
In this we have plotted the values of λ corresponding to orthonormal bases for which a precise estimate will be given in para. 4.1.2. We find that the minima are close to these values. A derivation in relation to λ (position of the base) discloses an extremum for the symmetrical bases but the orthogonal bases do not appear in any evident manner in the computation.

e) Let us note that the results of the simulation can be considered simply as qualitative in view of the low number of samples selected (300). A quantitative computation would require too much computer time with our present means. It will thus not be surprising to find an appreciable difference in the mean error between the proposed method and the results of simulation. The variation is, however, analogous as is shown by the comparison of diagrams 6 and 8 on the influence of the interval between interrogation.

4.1.2. - Condition of existence of orthonormal bases: Let us therefore attempt to find the conditions of existence of the orthonormal bases (and, when necessary, how we can better select the bases if this optimum condition cannot be obtained). Let us state $u = R_s/R$ and let $2/\mu$ be the spacing of the bases ($2/\mu$ will therefore be a multiple of $\text{OMEGA}_s P$). The reference for longitude is taken at μ of the meridian of the balloon and the base is located by θ : $(0x, 0s)$. We have

$$\vec{S_1 B} \cdot \vec{S_2 B} = 1 + u^2 \cos 2\mu - 2u \cos \gamma \cos \mu \cos \theta$$

The condition " $\alpha = \frac{\pi}{2}$ " is therefore written: $\cos \theta = \frac{1 - u^2 \cos 2\mu}{2u \cos \gamma \cos \mu}$



As orthonormal base therefore can exist only if

$$\frac{1 + u^2 \cos 2\mu}{2u \cos \psi \cos \mu} = 1$$

and therefore, if ψ less than ψ_0 , with

$$\cos \psi_0 = \frac{1 + u^2 \cos 2\mu}{2u \cos \mu}$$

The angle ψ_0 itself is defined only if

$$\frac{1 + u^2 \cos 2\mu}{2u \cos \mu} < 1$$

$$\mu > \mu_1 = \arccos \frac{1 + \sqrt{2u^2 - 1}}{2u} = \frac{\Omega_s T^*}{2}$$

In other words, for a satellite with known altitude (u and Ω_s), we therefore define:

- 1) a minimum interval of interrogation T^* defined by

$$T^* = \frac{2}{\Omega_s} \arccos \frac{1 + \sqrt{2u^2 - 1}}{2u}$$

An orthonormal base can exist only if the spacing of the bases is nT greater than T^* .

- 2) if this condition is verified, we define an angle such that

$$\psi_0 = \arccos \frac{1 + u^2 \cos 2\mu}{2u \cos \mu}$$

An orthonormal base exists only if ψ less than ψ_0 .

3) it is then located by

$$\cos \theta = \frac{\cos \varphi_0}{\cos \varphi}$$

4) if $\varphi = \varphi_0$, we find $\theta = 0$ at the limit. The bases are then symmetrical. If φ greater than φ_0 , it is no longer possible to find orthonormal bases. Then the best location is made possible by symmetrical bases.

Example: Let us take the case of an orbit at an altitude of 900 km: $u = 1.14062$; $\Omega_{\text{S}} = 0.0010123$ when we find $T^* = 231$ seconds.

In the case of the selected orbit, orthonormal bases can exist only if the time separating two positions of the satellite taken as bases is greater than 231 seconds. Since we took $T = 195$ seconds, we must therefore retain the positions $(n, n + 2)$ compatible with the conditions of visibility.

$$T = 390 \text{ sec.} \quad \varphi_0 = 23^\circ$$

$$T = 585 \text{ sec.} \quad \varphi_0 = 26^\circ 30'$$

$$T = 780 \text{ sec.} \quad \varphi_0 = 31^\circ$$

The result of this is that, if we find ourselves at a distance from the trace of less than 23° , we can select orthonormal bases in the case under consideration.

Note: Actually we should not tarry too much with these questions. We saw that the existence of orthonormal bases requires rather long intervals between interrogation. Now then, if we desire to limit the errors due to wind, we shall see subsequently that it is imperative to utilize shorter intervals between interrogation so that we no longer will have orthonormal bases. The symmetrical bases then represent the best compromise.

4.2 - Influence of Interval Between Interrogation

This influence was tested for symmetrical bases with a simulation from 300 samplings and the results plotted in diagrams 6 and 7. In regard to mean error (diagram 6), the results are close to those of diagram 8 obtained on the basis of the formulas of para. 4.1.1. We find an optimum interval between interrogation whose value increases with distance from the trace. However, the gain of accuracy beyond $T = 200$ seconds is no longer very large.

Diagram 7 concerns the probabilities of localization where we observe the same circumstances as previously. For large distances from the trace, we must select rather long T where $T = 400$ s. furnishes a probability of 98% at 20° . For short distance from the trace, an optimum interval T appears. At 5° from the trace, $T = 200$ s. furnishes a probability of 98%.

It should be noted that these conclusions are not conclusive because the presence of wind may decisively modify them. Large intervals between interrogation will result in a large displacement of the balloon and consequently produce errors of localization much higher than those exclusively due to error in measurement of distance.

4.3 - Influence of Distance from the Trace.

Diagram 10 furnishes the probabilities of localization at different distances from the trace for the same errors of distance and the same interval between interrogation. It should be noted that:

a) for symmetrical bases, the probability of localization increases with closeness to the trace (of. diagram 4). This corresponds to the best possible localization.

b) by contrast, the conclusions must be opposite if we consider the zones at the limit of visibility. For the initial base, the probability of localization increases with distance

from the trace.

It consequently appears that, if no data processing is carried out, i.e. if we accept all the bases, it is difficult to state any definite conclusions on the influence of the distance from the trace in regard to the probabilities of localization. Let us say simply that, the mean of probabilities is not very dependent on the distance from the trace. Although the number of simulating operations is somewhat low and more precise results would require a test on /IBM/ 7040, the variation of probability of localization at 1 km can be estimated as between 37 and 43% when the distance from the trace changes from 5° to 25° . Let us also point out that the results obtained seem to indicate that the probability of localization should pass through a maximum.

We should also keep in mind that, if we consider the mean errors given by the formula of para 4.1.1, we obtain the results plotted in diagram 5 which lead to the same conclusions as above.

4.4 - Influence of Typical Deviation.

Two cases corresponding to typical deviations respectively of 0.5 and 1.0 km were considered. The increase of the typical deviation from errors of distance brings about an appreciable reduction of the probabilities of localization. Diagram 11 illustrates this effect for a specific case and this effect seems as much larger at the distance from the trace is greater. If we double sigma, the probabilities of localization at 2 km become reduced by a ratio of approximately 1.6:2.

Localization error for symmetrical bases.

Orbit = 900 km; typical deviation = 0.5 km/hr.
Influence of interval between interrogation and of the distance from the trace.

Quantification of error in km:	T = 50 sec.	T = 100 sec.	T = 200 sec.	T = 400 sec.
1	76	138	174	170
2	97	134	122	118
3	77	26	4	12
4	27	2	0	0
5	17			
6	5			
7	1			
8	0			
9				
10				
>10				
Mean value of error in km:	1,90	1,06	0,87	0,94

Distance from the trace = 10^0

Quantification of error in km:	T = 50 sec.	T = 100 sec.	T = 200 sec.	T = 400 sec.
1	63	120	206	245
2	84	120	89	55
3	62	42	5	0
4	39	16	0	
5	18	2		
6	15	0		
7	13			
8	4			
9	0			
10	1			
>10				
Mean value of error in km.	2,47	1,33	0,77	0,67

Distance from the trace = 20°

Quantification of error in km.	T = 50	T = 100	T = 200	T = 400
1	45	94	150	236
2	50	67	106	63
3	29	71	36	1
4	24	40	7	0
5	42	10	1	
6	39	9	0	
7	19	2		
8	5	1		
10	14			
>10				
Mean value of error in km.	4,16	2,03	1,15	0,68

5. Conclusion

As already stated, the conclusions formulated here are incomplete. We took into account that the errors of localization due to electronic errors affect the measurements and are far from constituting the exclusive cause of inaccuracy. We shall therefore return subsequently to these problems which will be done in memorandum No. 9 which will be a summarization of memoranda 5, 6, 7 and 8.

However, we should already now insist in regard to the electronic errors:

a) on the influence of the position of the bases which is considerable since the areas of localization at the limit of visibility may be 10 to 20 times greater than for symmetrical bases.

b) on the influence of the typical deviation σ ; all things being equal, whenever the typical deviation is doubled, the error of localization also increases 100%.

In regard to the other factors, i.e. interval between interrogation and distance from the trace, the conclusions are even more incomplete because the influence of the wind (cf. memorandum No. 7) may lead to the opposite conclusions when the two factors are considered simultaneously.

For electronic errors, the error of localization is reduced when the interval between interrogation, starting from zero, increases to an optimum value which corresponds in principle to the orthonormal bases (but which actually depends on the statistical properties of the electronic errors). Except for short distances from the trace, since the optimum bases are obtained for long intervals between interrogation, the influence of the wind makes all of this again questionable (cf. memorandum no. 7 and 9).

For short intervals between interrogation, localization is better close to the trace. However, although this result would seem evident from the preceding paragraph, it will also need to be revised when the wind is taken into account.

Annex

Digital Generation of a Gauss White Noise.

In view of the fact that it is always possible to generate a Gauss spectrum noise $\text{PHI}(\omega)$ by passing a white noise through a linear filter, we were induced to examine the problem of the generation of a Gauss white noise of mean n and mean quadratic deviation / symbol missing/. This modification was made on the basis of the Marine Study E-570. (The procedure was suggested by a study of M.L. Guichard under contract DGRST No. 63 FR 226; Editions CERA, December 1965).

Resumé of Theory

Given six arbitrary numbers A_0, A_1, A_2, A_3 and A_4 , we generate a new sequence of values A_i^* as follows;

$$A_i^* = (A_i \oplus a) + A_{i+1} \quad (1)$$

where $\oplus$ signifies an addition without carry-over (exclusion) and where we agree to state $A_5 = A_0$. We thus obtain a sequence of pseudo-random numbers of uniform density. By adding A_i^* in steps of 5, we obtain a swquence of numbers with an approximately Gauss distribution (tending toward the normal law).

$$B_n = \sum_{i=0}^4 A_i^*$$

Programming Generation

We take the numbers a and A_1 between 0 and 1 and agree to carry out computation by fractions, i.e. to consider only numbers with a module of less than 1. Several practical problems then appear:

Cumulation in Calculating A_i^*

From formula (1), it will be seen that we may derive values of A_i^* of a module greater than 1 and will then have cumulation in the sign bit with consequent alteration of the value of A_i^* . In order to prevent this when such cumulation is found, the latter will be carried over to the bit of least weight and we will then have the following schema:

	sign	number
	0	1 0 0 0 0 0
	+ 0	1 0 0 0 0 0
	1	0 0 0
which gives	0	0 0 1

Adjustment of A_i^*

In order to calculate B_{eta} , A_i^* must be adjusted to prevent any cumulation. For this purpose, we eliminate the six digits of highest weight by an intersection with the binary number

0 , 0 0 0 0 0 0 1 1 1.

Obtaining the Mean m

The mean value of B_{eta} is equal to 5 times that of A_i^* after adjustment

$$\text{since } A_i^* = 2^{-6}/2 = 2^{-7}$$

$$\text{therefore } B_n = 5 \times 2^{-7} = 0,039062$$

In order to obtain the mean $\underline{m}$, we must therefore take

$$B = B - 0,039062 + m.$$

Adjusting mean quadratic deviation.

Experimentation furnished $\sigma = 0,009$. In order to obtain a deviation σ , we must therefore multiply $B_{\text{eta}}^* - m$ by 111 and will then have:

$$B_n^{**} = (B_n - m) \times 111 \sigma + m.$$

The numbers B_{eta} then constitute the desired sequence. Experimentation with 10,000 numbers of the correlation and density of probability has shown that this mode of generation is satisfactory.

Influence of Electronic Errors on Localization

```

PAF
i = 2 L D M A
i = 11 I
1 = read R Z E T
2 = print with 3 decimals TAB R TAB Z TAB E TAB T RC
3 = calculate  $V = \sqrt{(398599R^3/Z^3)}$ 
4 = calculate  $O = R(\cos E)^2 + (\sin E) \sqrt{(1-R^2(\cos E)^2)}$ 
5 = read F
6 = calculate  $Y = ZF$ 
7 = print with 0 decimals TAB Y RC TAB F RC
8 = calculate  $C = \arccos (O/(\cos F))$ 
9 = make  $L_1 = -C$ 
10 = calculate  $L_2 = L_1 + VT$ 
11 = calculate  $D_1 = \sqrt{(Z^2 + Z^2/R^2 - 2Z^2(\cos F)(\cos L_1)/R)}$ 
12 = calculate  $D_2 = \sqrt{(Z^2 + Z^2/R^2 - 2Z^2(\cos F)(\cos L_2)/R)}$ 
13 = state S=19839
14 = state (S)=0 k=1
15 = make S=S+1
16 = if S>19849 go to 19
17 = state (S)=0
18 = go to 15
19 = program 151,0
20 = state B= (19485)
21 = if variant, print with 6 decimals TAB B TAB
22 = calculate  $M_1 = D_1 + B$ 
23 = program 151,50
24 = state B=(19485)
25 = if variant, print with 6 decimals TAB B RC
26 = calculate  $M_2 = D_2 + B$ 
27 = calculate  $A_1 = R(Z^2 + Z^2/R^2 - M_1^2)/2Z^2$ 
28 = calculate  $A_2 = R(Z^2 + Z^2/R^2 - M_2^2)/2Z^2$ 
29 = Calculate  $G = \arctan \frac{(A_1(\cos L_2) - A_2(\cos L_1))}{(A_2(\sin L_1) - A_1(\sin L_2))}$ 
30 = calculate  $H = \arccos (A_1/(\cos (G - L_1)))$ 
31 = if variant, print with 6 decimals TAB G TAB H RC

```

- 33 -

54 = print with I decimals TAB K TAB I₁ RC TAB I₂ RC TAB I₃ RC
 TAB I₄ RC TAB I₅ RC TAB I₆ RC TAB I₇ RC TAB I₈ RC
 TAB I₉ RC TAB I₁₀ RC TAB I₁₁ RC RC
 55 = make L₁ = L₂
 56 = calculate L₂ = L₁ + VT
 57 = if L₂ > C go to 59
 58 = go to 11
 59 = go to 5
 END

Explanation of letter symbols.

R: " ρ " radius of balloon sphere / radius of orbit;
 Z: " R " radius of balloon sphere;
 E: e elevation;
 T: T time between interrogation;
 V: " W_0 " velocity of terrestrial rotation;
 F: " φ " distance from trace (in radians);
 Y: distance from trace (km);
 C: " 0 " 1/2 angle of visibility;
 D₁: " d₁ " L₁L₂: position of base;
 D₂: " d₂ "
 B: " η_1 " value of random variable;
 M₁, M₂: measured distances
 A₁, A₂: " D₁ " " D₂ "
 G: " $\wedge$ * "
 H: " φ "
 Q: error in km;
 I₁, I₂, ..., I₁₁: storage memories;
 P: no storage
 N: storage index;
 K: index;
 Programme 151,0: program 151,0
 Programme 151,50: program 151,50
 Declarations 34 to 47, clearing of error.

Subprogram of random-number generation

152,0M
0,5679588317 a
0,3869705200 A₀
0,0436897277 A₁
0,0375442504 A₂
0,9745445251 A₃
0,2219467163 A₄
0,0000000000 computer storage
0,0156249995
0,0390619998
102
FE
152,10F
0 mean
FE
152,11M
0,5 typical deviation
FE
152,15M
0,7247385143 a
0,1853297800 A₀
0,0921145366 A₁
0,2532481001 A₂
0,3326193517 A₃
0,2010415673 A₄
FE

151,0M
T12M152,12
A1EM152,0
A2EM152,1
T2M152,6
A15EVO
A3EVO

A2EM152,1+
V2TR1
A13M152,2+
A13V1D13
V13TW16384D12
T13M152,1+
I13TM152,7
A3R13
A15V1
V15TV5
RV6 Δ Y13
S3M152,8
M3M152,9
T13ER4
NF7R4
MF7M152,11
AF7M152,10
T7M152,29
RM152,12
FE

151,50M
T12M152,12
A1EM152,15
A2EM152,16
T2M152,21
A15EVO
A3EVO
A2EM152,16+
V2TR1A13M152,17+
A13V1D13
V13TW16384D13
T13M152,16+
I13TM152,7
A3R13

A15V1
 V15TV5
 RV6△Y13
 S3M152,8
 M3M152,9
 T13ER4
 NF7R4
 MF7M152,11
 AF7M152,10
 T7M152,29
 RM152,29
 FE

Existence and calculation of orthonormal bases.

1 = read R Z T
 2 = calculate $V = \sqrt{(398599R^3/Z^3)}$
 3 = calculate $D = \text{ARC COS } ((R + \sqrt{(2-R^2)})/2)$
 4 = calculate $M = 2D/V$
 5 = calculate $Q = VT/2$
 6 = print with 5 decimals TAB M TAB Q RC
 7 = if $T > M$ go to 1
 8 = calculate $B = (R^2 + (\text{COS } VT)) / (2R \text{ COS } (VT/2))$
 9 = calculate $G = \text{ARC COS } B$
 10 = print with 5 decimals TAB G RC
 11 = state $F = 0,087267$
 12 = if $F > G$ go to 17
 13 = calculate $L = \text{ARC COS } (B / \text{COS } F)$
 14 = print with 5 decimals TAB L RC
 15 = make $F = 2F$
 16 = go to 12
 END

T: T interval between interrogation
 R: R/R_s radius of balloon sphere / radius of orbit;
 Z: R radius of balloon sphere;
 V: " Ω_s " velocity of satellite rotation;
 M: " T^* "
 : " μ "
 G: " y_s " minimum distance from trace;
 F: " y " distance from trace;
 L: " θ " position of orthonormal base.

Diagram Captions

Diagram 1 - Influence of base position on probabilities of localization. a= probability of localization in %; b= P_x : probability of localization at x km; c= base position lambda in radian.

Diagram 2 - Influence of base position on probability of localization. a= probability in %; b= P_x : probability of establishing localization at better than x km; c= base position.

Diagram 3 - Probability of localization at less than 2 km as a function of base position for different distances from the trace. a= probability of localization at better than 2 km; b= base position in radian.

Diagram 4 - Influence of time between interrogation on errors of localization. a= error of localization in km; b= distance from trace = 5^0 ; c= position of base.

Diagram 5 - Influence of distance from trace. a= error of localization in km; b= base position in radian.

Diagram 6 - Errors of localization as a function of base position - justification of orthonormal bases. a= error of localization in km; b= locating orthonormal base; c= base position in radian.

Diagram 7 - Influence of time between interrogation and distance from the trace on the probabilities of localization. a= probability of localization at 2 km in %; b= interval between interrogation in sec.

Diagram 8 - Influence of interval between interrogation and of distance from trace in the mean error of localization (errors calculated from formula of para. 4.1.1.). a= mean error of localization in km; b= interval between interrogation $400T$ (sec).

Diagram 9 - Influence if interval between interrogation and of distance from trace on the errors of localization (sym-

metrical bases). a = mean error of localization in km;
 b = interval between interrogation T (sec).

Diagram 10 - /?/ /Photoreproduction fouled up/ b = distance from trace in degree.

Diagram 11 - Influence of typical deviation. Probability of localization at better than 2 km. a = probability of localization in %; b = distance from trace in degree.

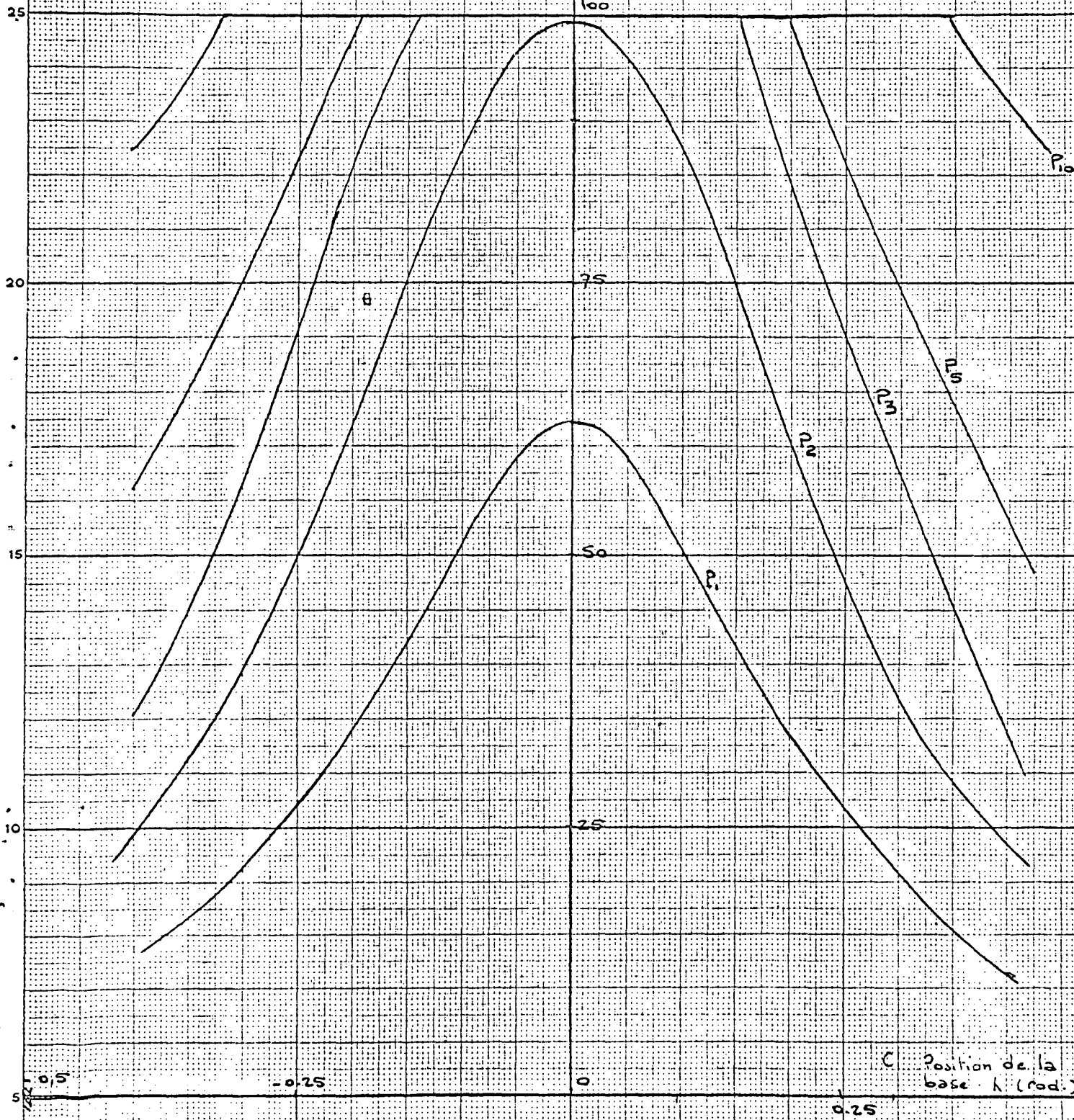
Planche I

Influence de la position des bases
sur les probabilités de localisation

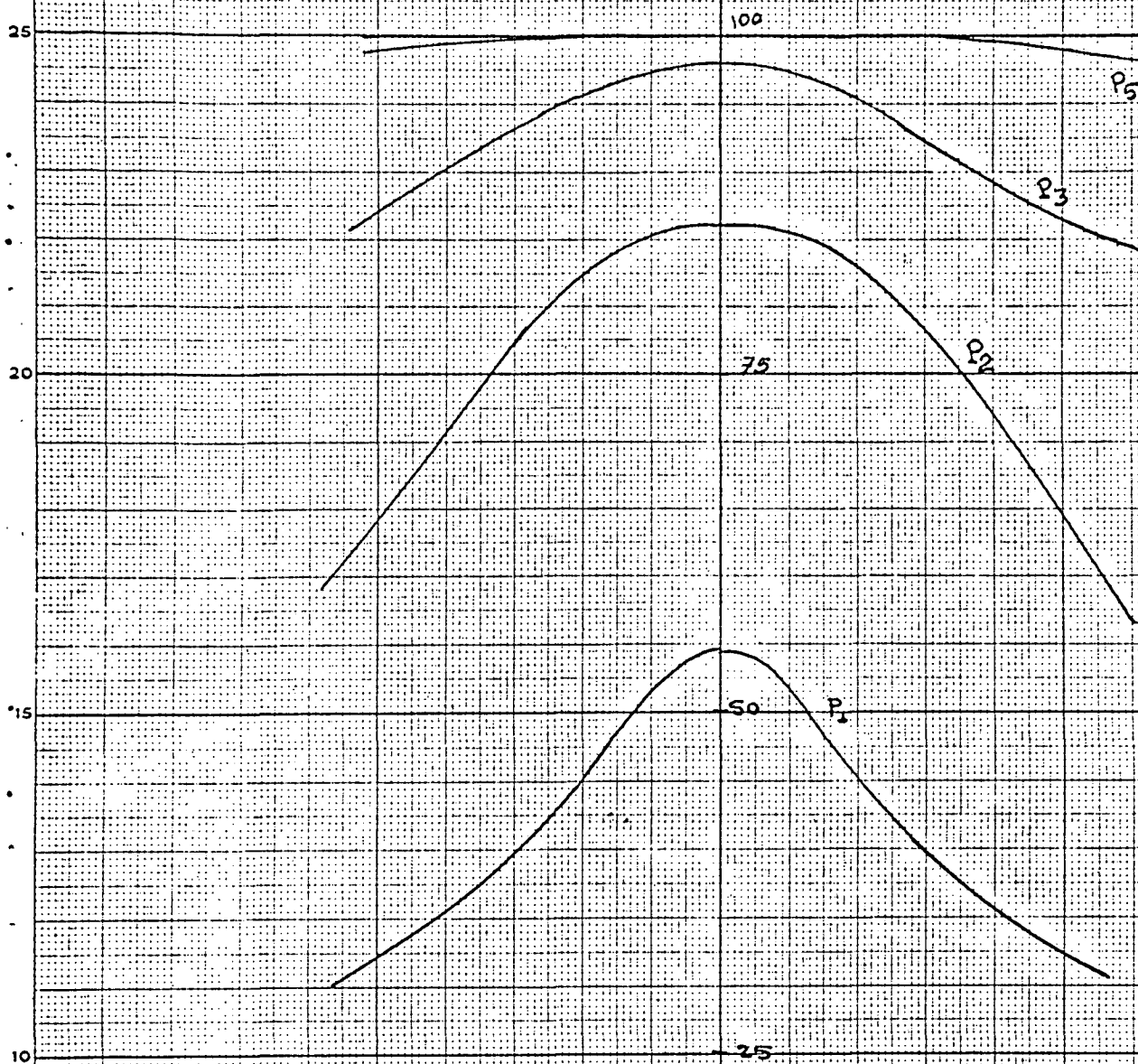
$\sigma = 0.5 \text{ km}$ $T = 1953$ $\varphi = 5^\circ$

a. Probabilité de localisation
(en %)

b. P_x Probabilité de localiser à x kilomètres:



42
a Probabilité %

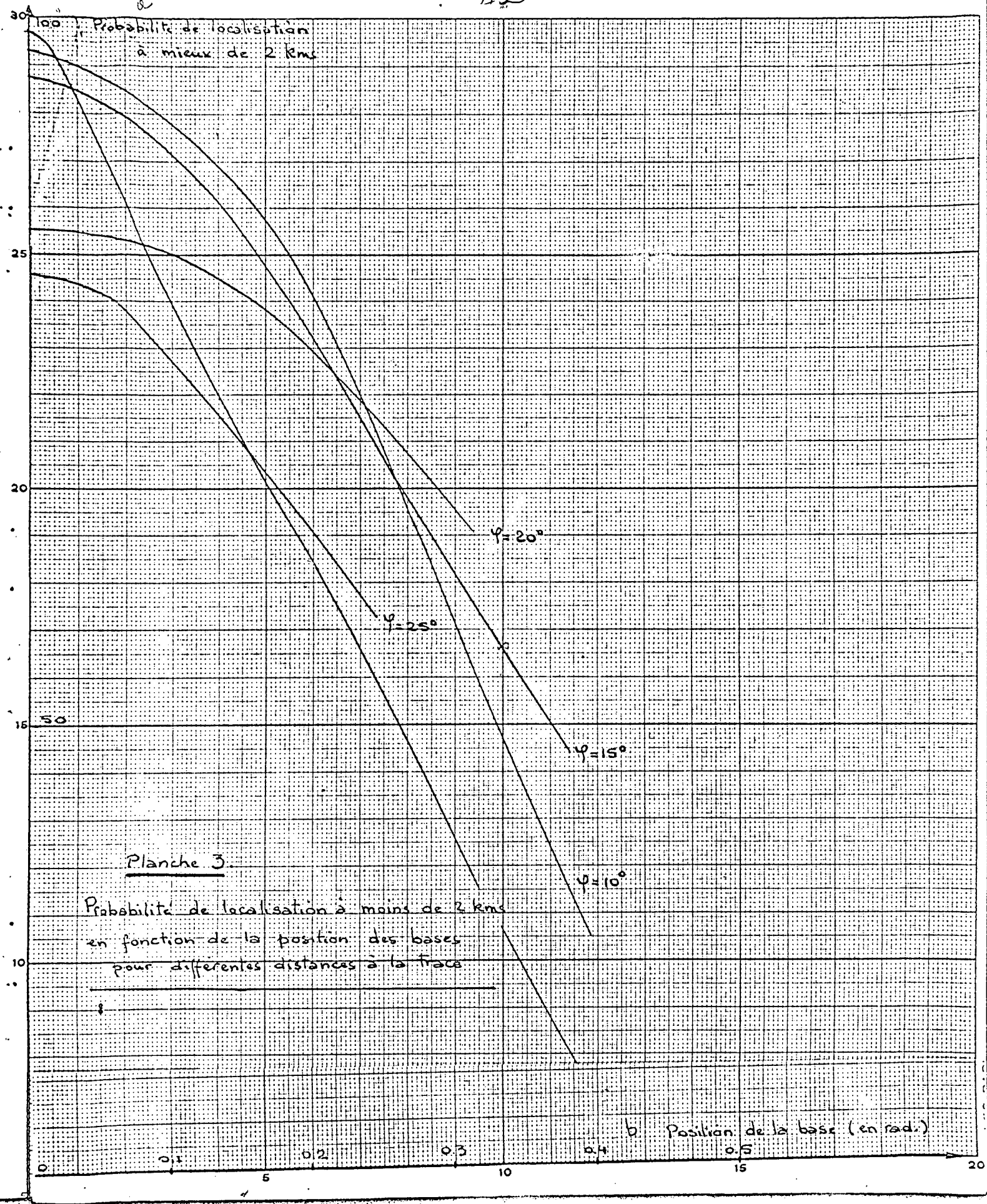


b P_x : Probabilité d'assurer
la localisation à mieux de 2 kilomètres

c Position de
la base
 h (rad)

Planche 2

Influence de la position des bases sur la probabilité
de localisation $T=195$ sec. $\sigma=0.5$ km. $\gamma=200$



Erreur de localisation
(en kms)

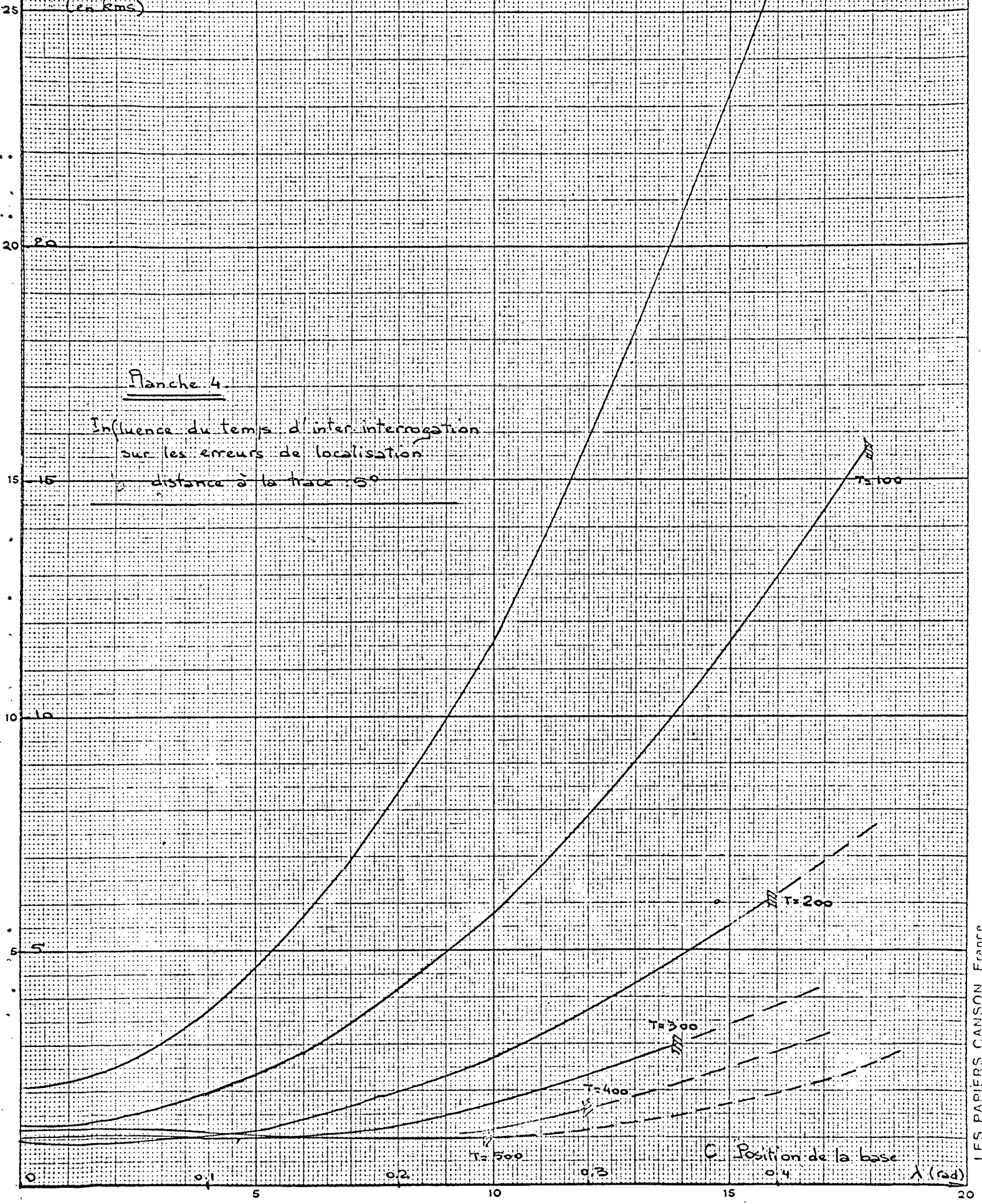
40

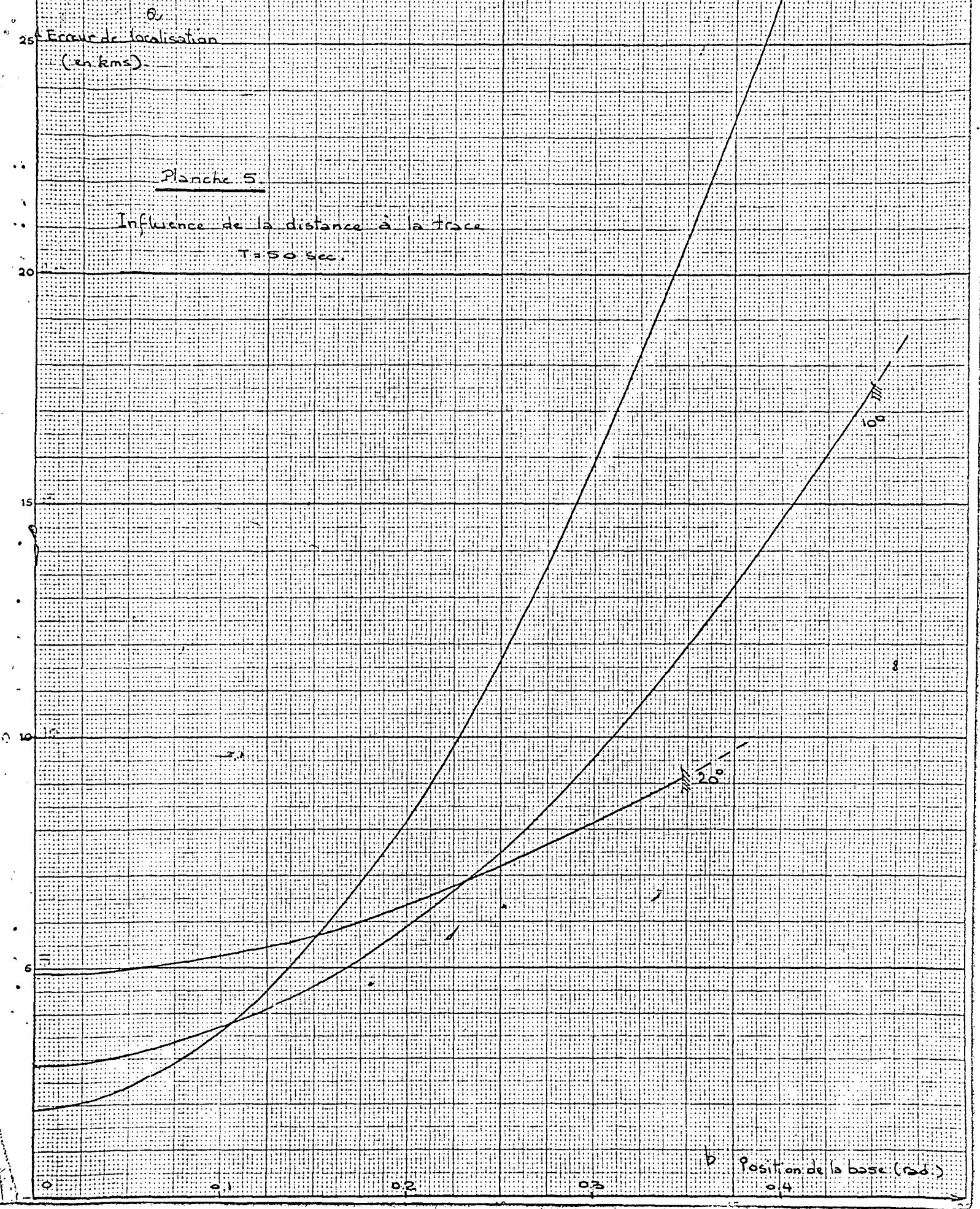
7.50

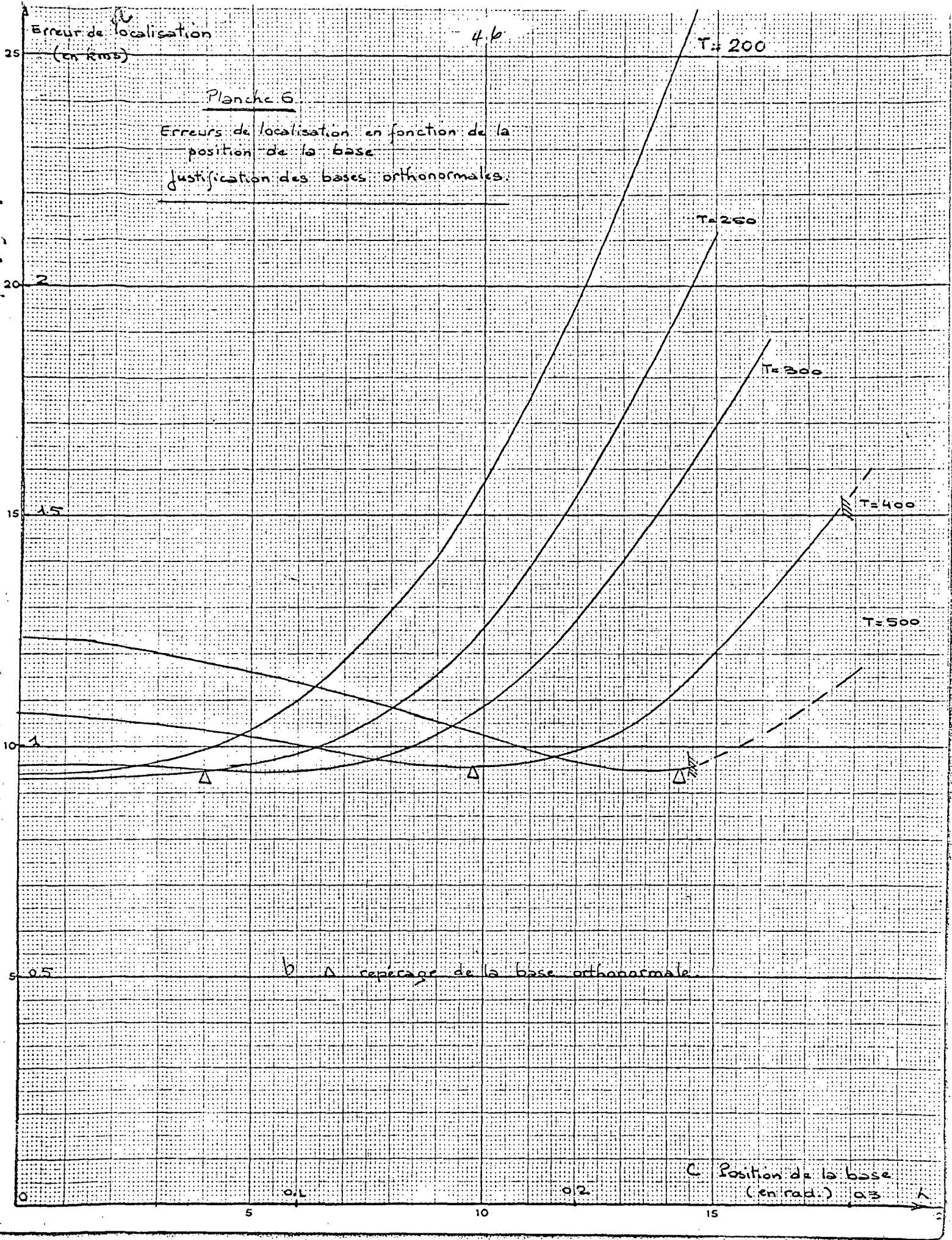
Planche 4

Influence du temps d'inter-interrogation
sur les erreurs de localisation

distance à la trace 50







a

Probabilité de localisation à 2 kms (en %)

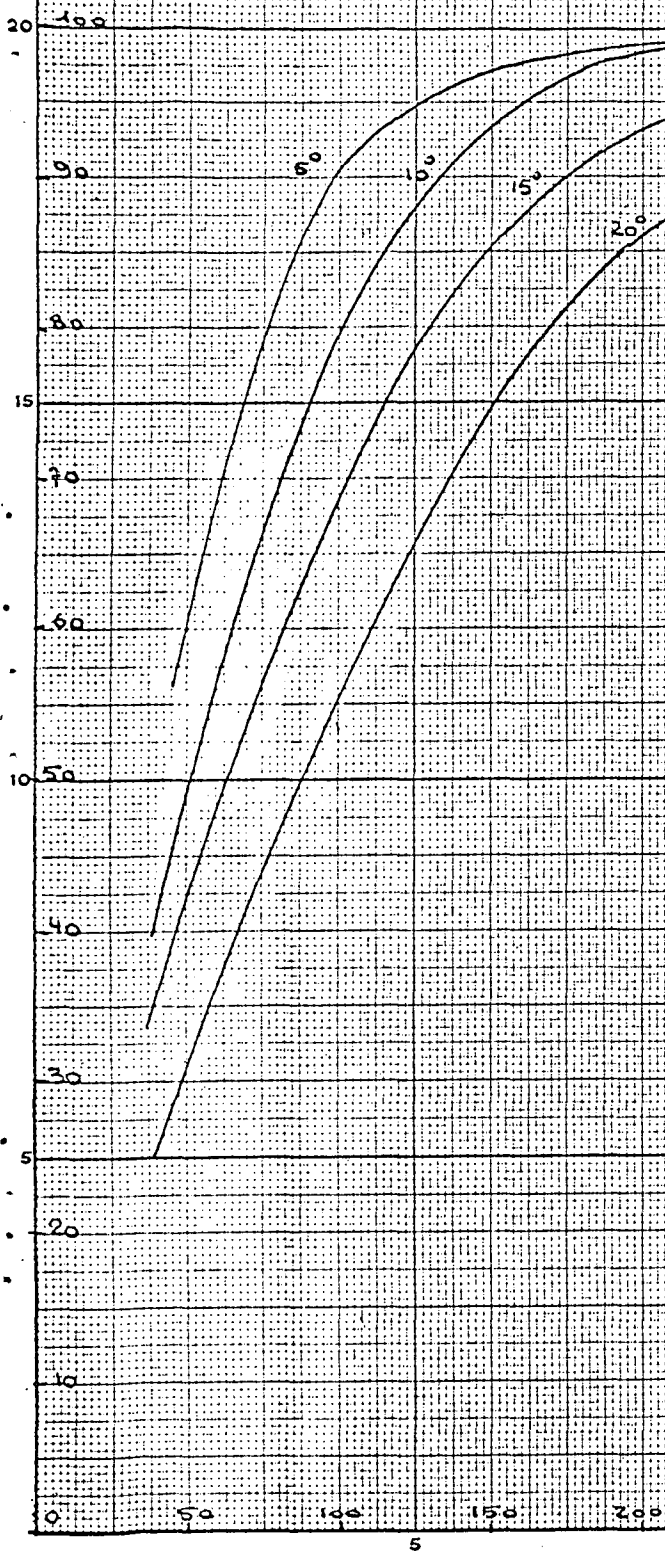


Planche 7

Influence du temps d'interrogation
et de la distance à la trace
sur les probabilités de localisation

b Temps d'interro-
(en sec.)

Planche 8

Influence du temps d'inter-interrogation
et de la distance à la trace sur
l'erreur moyenne de localisation

(erreurs calculées d'après la formule du § 4.1.1.)

A Erreur moyenne
de localisation
(km)

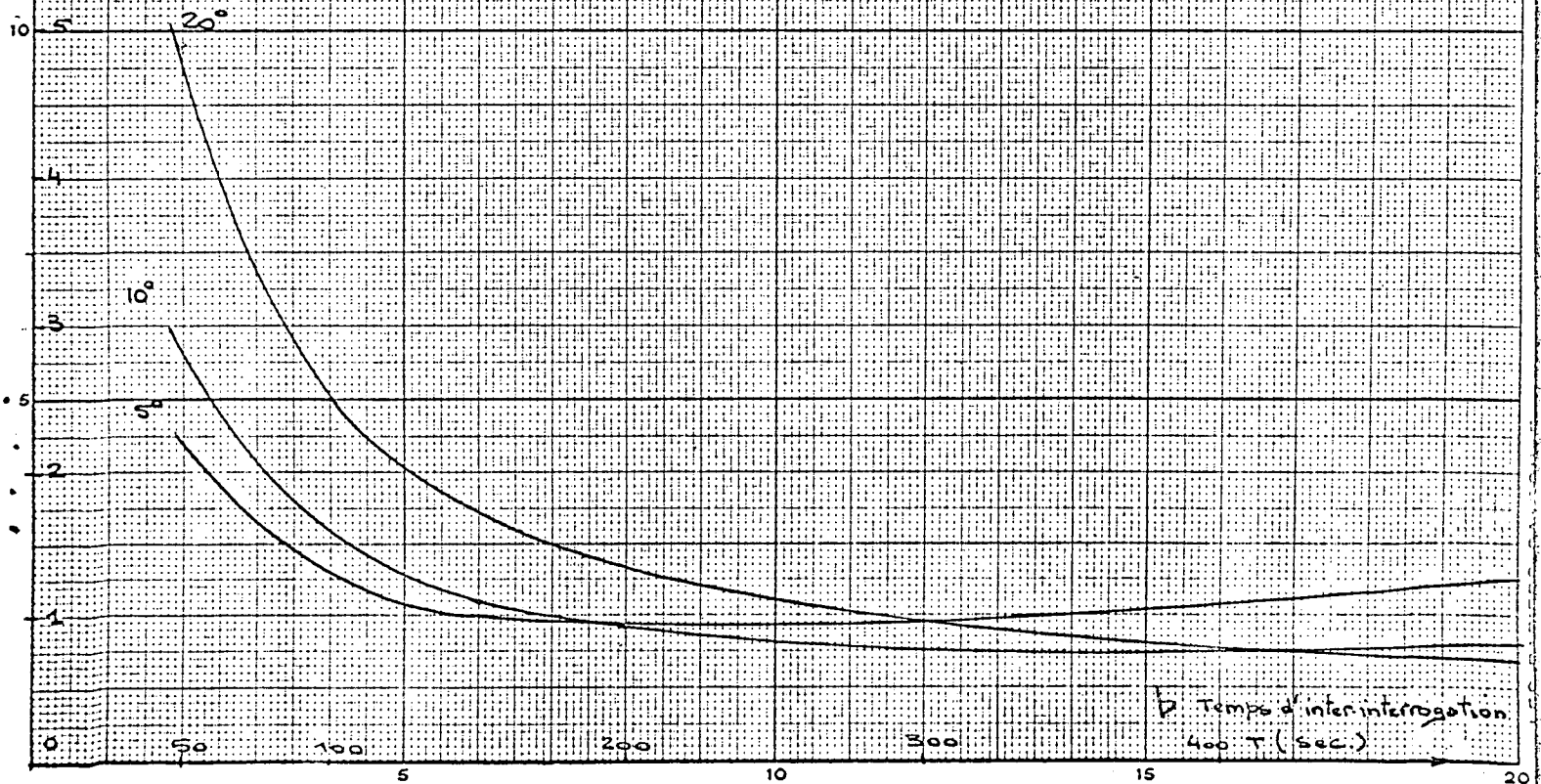
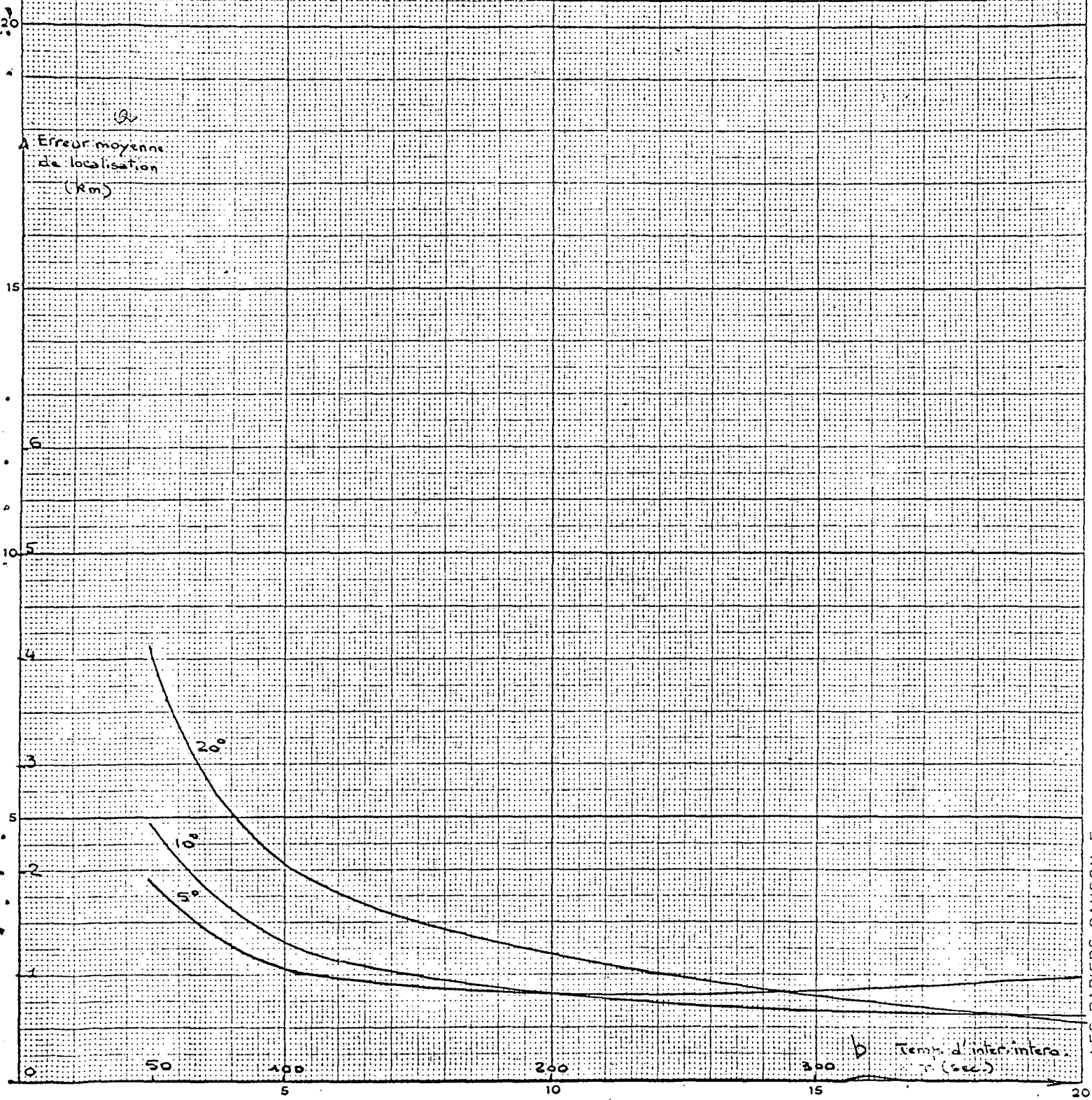


Planche 9

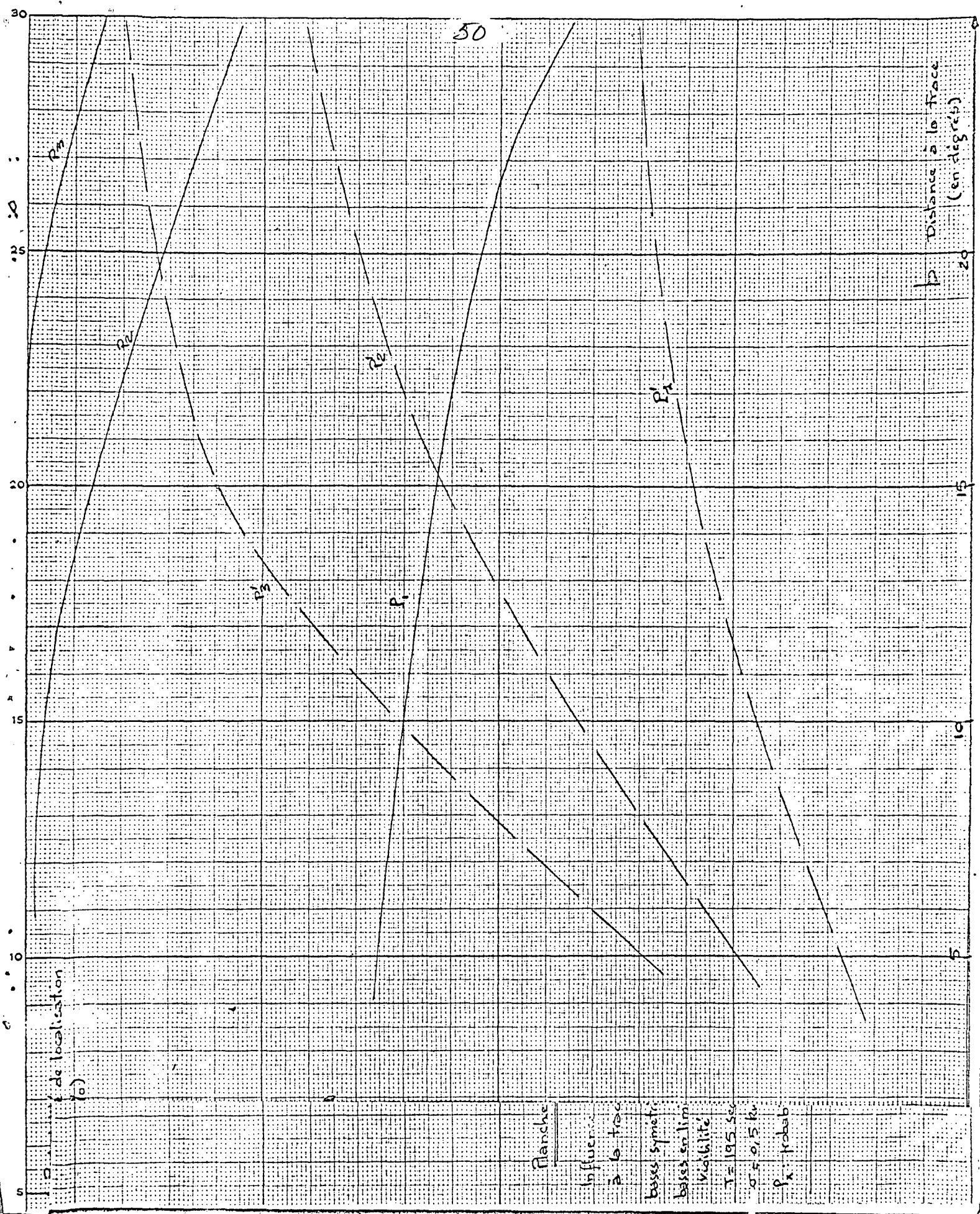
Influence du temps d'inter-interrogation
et de la distance à la trace sur les
erreurs de localisation.

(bases symétriques)

a Erreur moyenne
de localisation
(Km)



b Temp. d'inter-interro.
(sec)



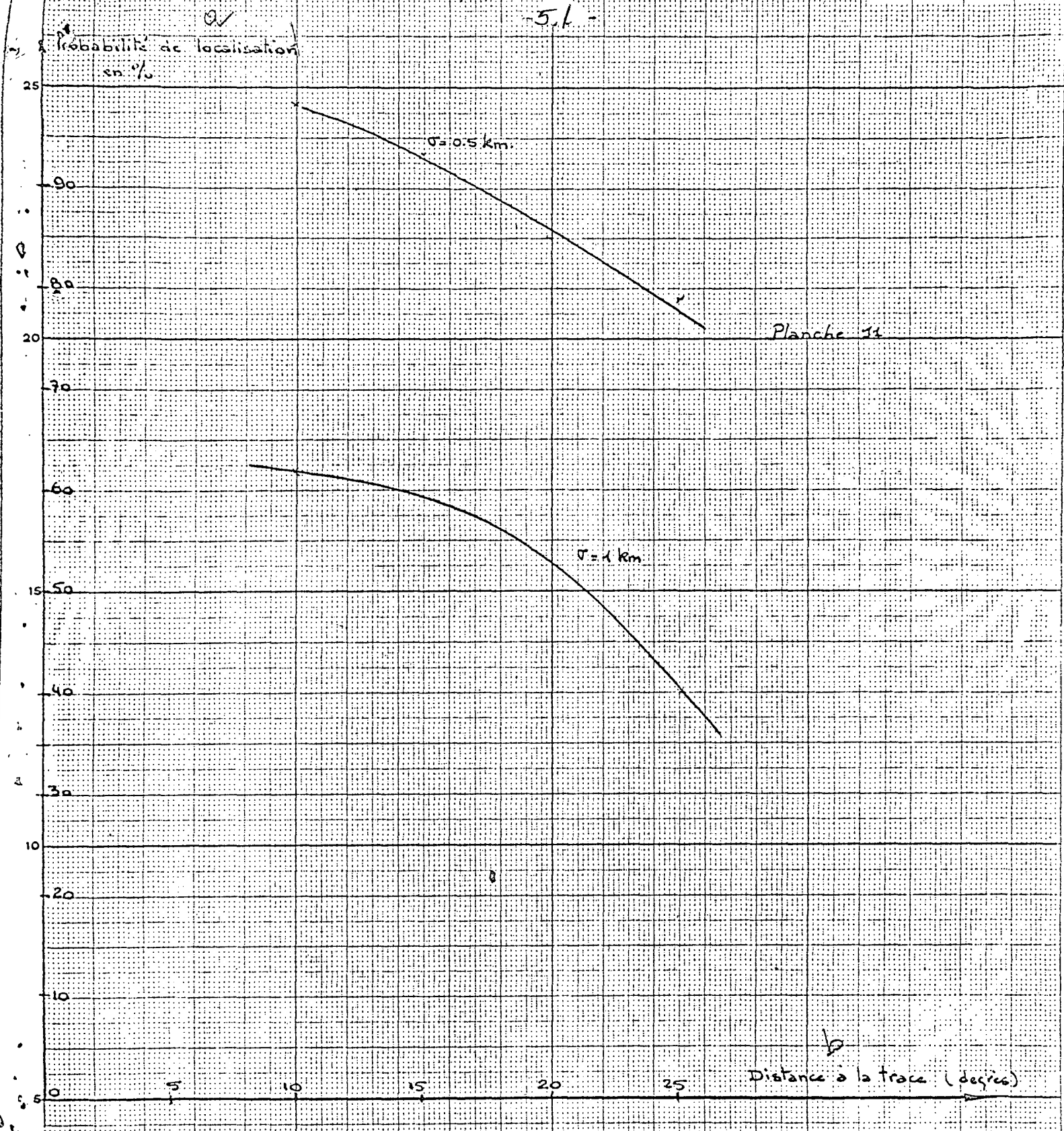


Planche 11

Planche 11. Influence de l'écart type. Probabilité pour une localisation à mieux de 2 kms.